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APPROXIMATING THE SOLUTION OF A FRACTIONAL ORDER MODEL OF NOVEL CORONAVIRUS (COVID-2019) UNDER CAPUTO–FABRIZIO DERIVATIVE

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ABSTRACT. This paper presents a fixed point iteration method for approximating the solution of a fractional order model of novel coronavirus (COVID-2019) under Caputo–Fabrizio derivative in Banach spaces. Our result is new and complements some existing results in the literature.

1. INTRODUCTION

Fixed Point Theory is concerned with solution of the equation

$$\ell = T\ell, \tag{1.1}$$

where T could be a nonlinear operator defined on a metric space. Any ℓ that solves (1.1) is called the fixed point of T and the collection all such elements is denoted by $F(T)$. Fixed point theory is an area in nonlinear analysis that has become very attractive and interesting with a large number of applications in various fields of mathematics and other branches of science. Fixed point theory has remained not only a field with a huge development, but also a very helpful means for solving various problems in different fields of mathematics. It is well known

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that fixed point theorems are used for proving the existence and uniqueness to various mathematical models like differential, integral and partial differential equations and variational inequalities etc., representing phenomena arising in different fields such as steady state temperature distribution, chemical equations, neutron transport theory, economic theories, epidemics and flow of fluids. Furthermore, it as also significant in the field of computer science, image processing, artificial intelligence, decision making, population dynamics, computer science, operational research, industrial engineering, pattern recognition, medicine, group health underwriting, management and many others.

Existence theorems are concerned with establishing sufficient conditions in which the equation (1.1) will have solution, but does not necessarily show how to find it. On the other hand, iteration method of fixed points is concerned with approximation or computation of sequences which converge to the solution of (1.1). When existence of a fixed point of an operator is guaranteed, obtaining constructive technique for finding such a fixed point is also paramount.

Very recently, Ofem et al. [24] introduced the following four steps iterative method for approximating the fixed points of almost contraction mappings and generalized α-nonexpansive mappings:

$$
\begin{cases}
\ell_0 \in \Lambda, \\
g_s = (1-\beta_s)\ell_s + \beta_s T\ell_s, \\
w_s = (1-\delta_s)T\ell_s + \delta_s T g_s, \qquad \forall s \geq 1, \\
\zeta_s = T w_s, \\
\ell_{s+1} = T\zeta_s,
\end{cases}
\tag{1.2}
$$

where $\{\delta_s\}$ and $\{\beta_s\}$ are sequences in (0,1).

Fractional Differential Equations (FDEs) involve fractional derivatives of the form $\frac{d^\nu}{dx^\nu}$, which are defined for $\nu > 0$, where ν is not necessarily an integer. They are generalization of the ordinary differential equations to a random (non-integer) order. Fractional differential equations have attracted much attentions due to their applications to model complex phenomena in engineering, physics, chemistry, biology and other fields (see [27,37,38]).

On the other hand, COVID-19 pandemic, also known as the coronavirus pandemic is an ongoing global pandemic of coronavirus disease 2019 (COVID-19) caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). The virus was first identified in December 2019 in Wuhan, China. The world Health Organization declared a Public Health Emergency of International Concern regarding COVID-19 on 30 January 2020, and later declared a pandemic on 11 March 2020. As of 26 April, more than 147 million cases have been confirmed, with 3.11 million death attributed to COVID-19, making it one of the deadliest pandemic in history.

Since the outbreak of COVID-19, several researchers have given special attentions and efforts to cure the deadly disease. Due to the significance of mathematical modeling, some coronavirus model have been introduced by Abdo et al. [1], Chen et al. [5], Hussain et al. [17] and Khan and Atangana [19].

Motivated by the above results, we will find the solution of fractional order model of novel coronavirus (COVID-2019) under Caputo–Fabrizio derivative using the efficient iterative method (1.2).

2. Preliminaries

The following definitions and lemmas will be useful in proving our main results.

Lemma 2.1. *[33] Let $\{\theta_s\}$ and $\{\lambda_s\}$ be nonnegative real sequences satisfying the following inequalities:*

$$\theta_{s+1} \leq (1-\sigma_s)\theta_s + \lambda_s,$$

where $\sigma_s \in (0,1)$ for all $s \in \mathbb{N}$, $\sum_{s=0}^{\infty} \sigma_s = \infty$ and $\lim_{s\to\infty} \frac{\lambda_s}{\sigma_s} = 0$, then $\lim_{s\to\infty} \theta_s = 0$.

Lemma 2.2. *[31] Let $\{\theta_s\}$ and $\{\lambda_s\}$ be nonnegative real sequences satisfying the following inequalities:*

$$\theta_{s+1} \leq (1-\sigma_s)\theta_s + \sigma_s\lambda_s,$$

where $\sigma_s \in (0,1)$ for all $s \in \mathbb{N}$, $\sum_{s=0}^{\infty} \sigma_s = \infty$ and $\lambda_s \geq 0$ for all $s \in \mathbb{N}$, then

$$0 \leq \limsup_{s\to\infty} \theta_s \leq \limsup_{s\to\infty} \lambda_s.$$

Definition 2.1. *[4] Let $\phi \in H(a,b)$, $b > a$, $a \in (-\infty, t)$ and $\nu \in [0,1]$, then the Caputo-Fabrizio derivative of order ν in the Caputo sense is given as*

$$^{CF}\mathfrak{D}^{\nu}\phi(t) = \frac{M(\nu)}{1-\nu}\int_a^t \phi'(n)\exp\left[-\frac{\nu(t-n)}{1-\nu}\right]dn, \tag{2.1}$$

where $M(\nu)$ is a normalization function such that $M(0) = M(1) = 1$.

The corresponding left fractional integral of ${}^{\mathrm{CF}}\mathcal{D}^{\nu}$ is defined in [2] as:

$$ {}^{\mathrm{CF}}\mathcal{I}^{\nu}\phi(t) = \frac{(1-\nu)}{M(\nu)}\phi(s) + \frac{\nu}{M(\nu)}\int_a^t \phi(n)dn. \tag{2.2} $$

Very recently, Hussain et al. [17] proposed a modified mathematical model of corona virus (COVID-19) as follows:

$$
\begin{aligned}
{}^{\mathrm{CF}}\mathcal{D}^{\nu}\mathcal{S}_p &= \bigwedge_p - \mu_p\mathcal{S}_p - \frac{\eta_p\mathcal{S}_p(\mathcal{I}_p + \Psi\mathscr{A}_p)}{\mathscr{N}_p} - \eta_w\mathcal{S}_p\mathscr{M},\\
{}^{\mathrm{CF}}\mathcal{D}^{\nu}\mathscr{E}_p &= \frac{\eta_p\mathcal{S}_p(\mathcal{I}_p + \Psi\mathscr{A}_p)}{\mathscr{N}_p} + \eta_w\mathcal{S}_p\mathscr{M} - (1-\vartheta_p)\omega_p\mathscr{E}_p - \vartheta_p\wp_p\mathscr{E}_p - \mu_p\mathscr{E}_p,\\
{}^{\mathrm{CF}}\mathcal{D}^{\nu}\mathcal{I}_p &= (1-\vartheta_p)\omega_p\mathscr{E}_p - (\tau_p + \mu_p)\mathcal{I}_p,\\
{}^{\mathrm{CF}}\mathcal{D}^{\nu}\mathscr{A}_p &= \vartheta_p\wp_p\mathscr{E}_p - (\tau_{ap} + \mu_p)\mathscr{A}_p,\\
{}^{\mathrm{CF}}\mathcal{D}^{\nu}\mathscr{R}_p &= \tau_p\mathcal{I}_p + \tau_{ap}\mathscr{A}_p - \mu_p\mathscr{R}_p, \qquad (2.3)\\
{}^{\mathrm{CF}}\mathcal{D}^{\nu}\mathscr{M}_p &= \beta_p\mathcal{I}_p + \sigma_p\mathscr{A}_p - \rho\mathscr{M},
\end{aligned}
$$

where ν denotes the fractional order parameter and the model variables in (2.3) are nonnegative and the initial conditions are defined as:

$$
\begin{aligned}
\mathcal{S}_p(0) &= \mathcal{S}_p(0) \geq 0, \quad \mathscr{E}_p(0) = \mathscr{E}_p(0) \geq 0, \quad \mathcal{I}_p(0) = \mathcal{I}_p(0) \geq 0\\
\mathscr{A}_p(0) &= \mathscr{A}_p(0) \geq 0, \quad \mathscr{R}_p(0) = \mathscr{R}_p(0) \geq 0, \quad \mathscr{M}_p(0) = \mathscr{I}_p(0) \geq 0.
\end{aligned}
$$

The model (2.3) can be re-written in the following form:

$$
\begin{cases}
{}^{\mathrm{CF}}\mathcal{D}^{\nu}\psi(t) = \mathscr{U}(t, \psi(t)),\\
\psi(0) = \psi_0, \quad 0 < t < T < \infty,
\end{cases} \tag{2.4}
$$

where the vector $\psi(t) = (\mathcal{S}_p, \mathscr{E}_p, \mathcal{I}_p, \mathscr{A}_p, \mathscr{R}_p, \mathscr{M}_p)$ and $\mathscr{U}$ in (2.4) stand for the state variables and a continuous vector function respectively defined as:

$$
\mathscr{U} = \begin{bmatrix} \mathscr{U}_1 \\ \mathscr{U}_2 \\ \mathscr{U}_3 \\ \mathscr{U}_4 \\ \mathscr{U}_5 \\ \mathscr{U}_6 \end{bmatrix}
$$

$$= \begin{bmatrix} \bigwedge_p -\mu_p \mathcal{S}_p(t) - \frac{\eta_p \mathcal{S}_p(t)(\mathcal{I}_p(t)+\Psi\mathscr{A}_p(t)}{\mathscr{N}_p(t)} - \eta_w \mathcal{S}_p(t)\mathscr{M}(t) \\ \frac{\eta_p \mathcal{S}_p(t)(\mathcal{I}_p(t)+\Psi\mathscr{A}_p(t)}{\mathscr{N}_p(t)} + \eta_w \mathcal{S}_p(t)\mathscr{M}(t) - (1-\vartheta_p)\omega_p\mathscr{E}_p(t) - \vartheta_p\wp_p\mathscr{E}_p(t) - \mu_p\mathscr{E}_p(t) \\ (1-\vartheta_p)\omega_p\mathscr{E}_p(t) - (\tau_p+\mu_p)\mathcal{I}_p(t) \\ \vartheta_p\wp_p\mathscr{E}_p(t) - (\tau_{ap}+\mu_p)\mathscr{A}_p(t) \\ \tau_p\mathcal{I}_p(t) + \tau_{ap}\mathscr{A}_p(t) - \mu_p\mathscr{R}_p(t) \\ \beta_p\mathcal{I}_p(t) + \sigma_p\mathscr{A}_p(t) - \rho\mathscr{M}(t) \end{bmatrix}$$

with the initial conditions $\psi_0(t) = (\mathcal{S}_p(0), \mathscr{E}_p(0), \mathcal{I}_p(0), \mathscr{A}_p(0), \mathscr{R}_p(0), \mathscr{M}_p(0))$.

The problem (2.4) can be reformulated in the following integral equation [17]:

$$\psi(t) = \psi_0 + \mathscr{F}(\nu)\mathscr{U}(t, \psi(t)) + \mathscr{W}(\nu)\int_a^t \mathscr{U}(\ell, \psi(\ell))d\ell, \tag{2.5}$$

where $\mathscr{F}(\nu) = \frac{1-\nu}{M(\nu)}$ and $\mathscr{W}(\nu) = \frac{\nu}{M(\nu)}$.

Let $0 \le t \le T$, we define a Banach space by using $\mathfrak{J} = [0, T]$ as $\mathscr{G} = (\mathfrak{J}, \mathbb{R}^6)$ under the supremum norm given by

$$\|\psi\| = \sup_{t\in\mathfrak{J}}\{|\psi(t)| : \psi \in \mathscr{G}\}.$$

Theorem 2.1. *[see [17]] We assume that the following conditions are satisfied:*

(C_1) *There exists a constant $L_{\mathscr{U}} > 0$ such that*

$$|\mathscr{U}(t, \psi_1(t)) - \mathscr{U}(t, \psi_2(t))| \le L_{\mathscr{U}}|\psi_1 - \psi_2|, \text{ for each } \psi \in \mathscr{G} \text{ and } t \in [0, T]$$

(C_2) $(\mathscr{F}(\nu) + T\mathscr{F}(\nu))L_{\mathscr{U}} < 1$.

Then (2.4) *has a unique solution.*

3. Main result

In this section, we approximate the solution of problem (2.4) by utilizing the iterative method (1.2).

Now we present our main result in this section as follows:

Theorem 3.1. *Suppose that all conditions (C_1)—(C_2) in Theorem 2.1 are fulfilled. Let $\delta_s, \beta_s \in [0, 1]$ be sequences of the iteration process* (1.2) *such that $\sum_{s=0}^{\infty} \delta_s\beta_s = \infty$. Then the problem* (2.4) *has a solution, say z and the iteration process* (1.2) *converges to z.*

Proof. We consider the Banach space $\mathscr{G} = (\mathfrak{J}, \mathbb{R}^6)$ under the supremum norm given by

$$\|\psi\| = \sup_{t \in \mathfrak{J}} \{|\psi(t)| : \psi \in \mathscr{G}\}.$$

Let $\{\ell_s\}$ be an iterative sequence generated by the iterative algorithm (1.2) for the operator $\mathcal{A} : \mathscr{G} \to \mathscr{G}$ defined by

$$\mathcal{A}\psi(t) = \psi_0 + \mathscr{F}(\nu)\mathscr{U}(t, \psi(t)) + \mathscr{W}(\nu) \int_a^t \mathscr{U}(\ell, \psi(\ell)) d\ell. \tag{3.1}$$

We will show that $\ell_s \to z$ as $s \to \infty$.

From (1.2), (3.1) and the assumptions (C_1)–(C_2) we have

$$\begin{aligned}
\|g_s - z\| &= \|(1-\beta_s)\ell_s + \beta_s \mathcal{A}\ell_s - z\| \\
&\leq (1-\beta_n) \max_{t \in [0,T]} |\ell_s(t) - z(t)| + \beta_n \max_{t \in [0,T]} |\mathcal{A}\ell_s(t) - \mathcal{A}z(t)| \\
&= (1-\beta_s) \max_{t \in [0,T]} |\ell_s(t) - z(t)| \\
&\quad + \beta_s \max_{t \in [0,T]} \Big|\psi_0 + \mathscr{F}(\nu)\mathscr{U}(t, \ell_s(t)) + \mathscr{W}(\nu) \int_a^t \mathscr{U}(\ell, \ell_s(\ell)) d\ell \\
&\quad - (\psi_0 + \mathscr{F}(\nu)\mathscr{U}(t, z(t)) + \mathscr{W}(\nu) \int_a^t \mathscr{U}(\ell, z(\ell)) d\ell)\Big| \\
&= (1-\beta_s) \max_{t \in [0,T]} |\ell_s(t) - z(t)| \\
&\quad + \beta_s \max_{t \in [0,T]} |\mathscr{F}(\nu)(\mathscr{U}(\ell, \ell_s(t)) - \mathscr{U}(t, z(t))) \\
&\quad + \mathscr{W}(\nu) \int_a^t (\mathscr{U}(\ell, \ell_s(\ell)) - \mathscr{U}(\ell, z(\ell))) d\ell| \\
&\leq (1-\beta_s) \max_{t \in [0,T]} |\ell_s(t) - z(t)| \\
&\quad + \beta_s [\mathscr{F}(\nu) \max_{t \in [0,T]} |\mathscr{U}(t, \ell_s(t)) - \mathscr{U}(t, z(t))| \\
&\quad + \mathscr{W}(\nu) \max_{t \in [0,T]} \int_a^t |(\mathscr{U}(\ell, \ell_s(\ell)) - \mathscr{U}(\ell, z(\ell))) d\ell|] \\
&\leq (1-\beta_s) \max_{t \in [0,T]} |\ell_s(t) - z(t)| \\
&\quad + \beta_n [\mathscr{F}(\nu) L_{\mathscr{U}} \max_{t \in [0,T]} |\ell_s(t)) - z(t)| \\
&\quad + \mathscr{W}(\nu) L_{\mathscr{U}} \max_{t \in [0,T]} \int_a^t |\ell_s(\ell) - z(\ell)| d\ell]
\end{aligned}$$

$$
\begin{aligned}
&\leq (1-\beta_s)\|\ell_s - z\| \\
&\quad +\beta_s[\mathscr{F}(\nu)+T\mathscr{W}(\nu)]L_{\mathscr{U}}\|\ell_s - z\| \qquad (3.2)\\
&= \{1-\beta_s(1-[\mathscr{F}(\nu)+T\mathscr{W}(\nu)L_{\mathscr{U}}])\}\|\ell_s - z\|. \qquad (3.3)
\end{aligned}
$$

$$
\begin{aligned}
\|w_s - z\| &= \|(1-\delta_s)\mathcal{A}\ell_s + \delta_s\mathcal{A}g_n - z\| \\
&\leq (1-\delta_s)\max_{t\in[0,T]}|\mathcal{A}\ell_s(t)-\mathcal{A}z(t)| + \delta_s\max_{t\in[0,T]}|\mathcal{A}g_s(t)-\mathcal{A}z(t)| \\
&= (1-\delta_s)\max_{t\in[0,T]}\Big|\psi_0+\mathscr{F}(\nu)\mathscr{U}(t,\ell_s(t))+\mathscr{W}(\nu)\int_a^t\mathscr{U}(\ell,\ell_s(\ell))d\ell \\
&\quad -(\psi_0+\mathscr{F}(\nu)\mathscr{U}(t,z(t))+\mathscr{W}(\nu)\int_a^t\mathscr{U}(\ell,z(\ell))d\ell)\Big| \\
&\quad +\delta_s\max_{t\in[0,T]}\Big|\psi_0+\mathscr{F}(\nu)\mathscr{U}(t,g_s(t))+\mathscr{W}(\nu)\int_a^t\mathscr{U}(\ell,g_s(\ell))d\ell \\
&\quad -(\psi_0+\mathscr{F}(\nu)\mathscr{U}(t,z(t))+\mathscr{W}(\nu)\int_a^t\mathscr{U}(\ell,z(\ell))d\ell)\Big| \\
&= (1-\delta_s)\max_{t\in[0,T]}|\mathscr{F}(\nu)(\mathscr{U}(t,\ell_s(t))-\mathscr{U}(t,z(t))) \\
&\quad +\mathscr{W}(\nu)\int_a^t(\mathscr{U}(\ell,\ell_s(\ell))-\mathscr{U}(\ell,z(\ell)))d\ell| \\
&\quad +\delta_s\max_{t\in[0,T]}|\mathscr{F}(\nu)(\mathscr{U}(t,g_s(t))-\mathscr{U}(t,z(t))) \\
&\quad +\mathscr{W}(\nu)\int_a^t(\mathscr{U}(\ell\, g_s(\ell))-\mathscr{U}(\ell,z(\ell)))d\ell| \\
&\leq (1-\delta_s)[\mathscr{F}(\nu)\max_{t\in[0,T]}|\mathscr{U}(t,\ell_s(t))-\mathscr{U}(t,z(t))| \\
&\quad +\mathscr{W}(\nu)\max_{t\in[0,T]}\int_a^t|(\mathscr{U}(\ell,\ell_s(\ell))-\mathscr{U}(\ell,z(\ell)))d\ell|] \\
&\quad +\delta_s[\mathscr{F}(\nu)\max_{t\in[0,T]}|\mathscr{U}(t,g_s(t))-\mathscr{U}(t,z(t))| \\
&\quad +\mathscr{W}(\nu)\max_{t\in[0,T]}\int_a^t|(\mathscr{U}(\ell,g_s(\ell))-\mathscr{U}(\ell,z(\ell)))d\ell|] \\
&\leq (1-\delta_s)[\mathscr{F}(\nu)L_{\mathscr{U}}\max_{t\in[0,T]}|\ell_s(t))-q(t)| \\
&\quad +\mathscr{W}(\nu)L_{\mathscr{U}}\max_{t\in[0,T]}\int_a^t|\ell_s(\ell)-z(\ell)|d\ell] \\
&\quad +\delta_s[\mathscr{F}(\nu)L_{\mathscr{U}}\max_{t\in[0,T]}|g_s(t))-z(t)|
\end{aligned}
$$

$$
\begin{aligned}
& \quad +\mathscr{W}(\nu)L_{\mathscr{U}} \max_{t\in[0,T]} \int_a^t |g_s(\ell) - z(\ell)|d\ell] \\
& \leq \quad (1-\delta_s)[\mathscr{F}(\nu) + T\mathscr{W}(\nu)L_{\mathscr{U}}]\|\ell_s - z\| \\
& \quad +\delta_s[\mathscr{F}(\nu) + T\mathscr{W}(\nu)L_{\mathscr{U}}]\|g_s - z\|.
\end{aligned} \tag{3.4}
$$

$$
\begin{aligned}
\|\zeta_s - z\| &= \|\mathcal{A}w_s - z\| = \max_{t\in[0,T]} |\mathcal{A}w_n(t) - \mathcal{A}z(t)| \\
&= \max_{t\in[0,T]} \Big|\psi_0 + \mathscr{F}(\nu)\mathscr{U}(t, w_s(t)) + \mathscr{W}(\nu)\int_a^t \mathscr{U}(\ell, w_s(\ell))d\ell \\
&\quad -(\psi_0 + \mathscr{F}(\nu)\mathscr{U}(t, z(t)) + \mathscr{W}(\nu)\int_a^t \mathscr{U}(\ell, z(\ell))d\ell)\Big| \\
&= \max_{t\in[0,T]} |\mathscr{F}(\nu)(\mathscr{U}(t, w_s(t)) - \mathscr{U}(t, z(t))) \\
&\quad +\mathscr{W}(\nu)\int_a^t (\mathscr{U}(\ell, w_s(\ell)) - \mathscr{U}(\ell, z(\ell)))d\ell| \\
&\leq \mathscr{F}(\nu)\max_{t\in[0,T]} |\mathscr{U}(t, w_s(t)) - \mathscr{U}(t, z(t))| \\
&\quad +\mathscr{W}(\nu)\max_{t\in[0,T]} \int_a^t |(\mathscr{U}(\ell, w_s(\ell)) - \mathscr{U}(\ell, z(\ell)))d\ell| \\
&\leq \mathscr{F}(\nu)L_{\mathscr{U}} \max_{t\in[0,T]} |w_s(t)) - z(t)| \\
&\quad +\mathscr{W}(\nu)L_{\mathscr{U}} \max_{t\in[0,T]} \int_a^t |w_s(\ell) - z(\ell)|d\ell \\
&\leq [\mathscr{F}(\nu) + T\mathscr{W}(\nu)L_{\mathscr{U}}]\|w_s - z\|.
\end{aligned} \tag{3.5}
$$

$$
\begin{aligned}
\|\ell_{s+1} - z\| &= \|\mathcal{A}\zeta_s - z\| \\
&= \max_{t\in[0,T]} |\mathcal{A}\zeta_s(t) - \mathcal{A}z(t)| \\
&= \max_{t\in[0,T]} \Big|\psi_0 + \mathscr{F}(\nu)\mathscr{U}(t, y_n(t)) + \mathscr{W}(\nu)\int_a^t \mathscr{U}(\ell, \zeta_s(\ell))d\ell \\
&\quad -(\psi_0 + \mathscr{F}(\nu)\mathscr{U}(t, z(t)) + \mathscr{W}(\nu)\int_a^t \mathscr{U}(\ell, z(\ell))d\ell)\Big| \\
&= \max_{t\in[0,T]} |\mathscr{F}(\nu)(\mathscr{U}(t, \zeta_s(t)) - \mathscr{U}(t, z(t))) \\
&\quad +\mathscr{W}(\nu)\int_a^t (\mathscr{U}(\ell, \zeta_s(\ell)) - \mathscr{U}(\ell, z(\ell)))d\ell|
\end{aligned}
$$

$$
\begin{aligned}
&\leq \mathscr{F}(\nu) \max_{t\in[0,T]} |\mathscr{U}(t,\zeta_s(t)) - \mathscr{U}(t,z(t))| \\
&\quad + \mathscr{W}(\nu) \max_{t\in[0,T]} \int_a^t |(\mathscr{U}(x,\zeta_s(\ell)) - \mathscr{U}(\ell,z(\ell)))d\ell| \tag{3.6}\\
&\leq \mathscr{F}(\nu) L_{\mathscr{U}} \max_{t\in[0,T]} |\zeta_s(t)) - z(t)| \\
&\quad + \mathscr{W}(\nu) L_{\mathscr{U}} \max_{t\in[0,T]} \int_a^t |\zeta_s(\ell) - z(\ell)| d\ell \\
&\leq [\mathscr{F}(\nu) + T\mathscr{W}(\nu) L_{\mathscr{U}}] \|\zeta_s - z\|. \tag{3.7}
\end{aligned}
$$

Using (3.4), (3.5), (3.6) and (3.7), we obtain

$$
\begin{aligned}
\|\ell_{s+1} - z\| &\leq [\mathscr{F}(\nu) + T\mathscr{W}(\nu) L_{\mathscr{U}}]^3 \\
&\quad \times \{1 - \delta_s \beta_s (1 - [\mathscr{F}(\nu) + T\mathscr{W}(\nu) L_{\mathscr{U}}])\} \|\ell_s - z\|. \tag{3.8}
\end{aligned}
$$

From assumption (C_2), (3.8) reduces into

$$
\|\ell_{s+1} - q\| \leq \{1 - \delta_s \beta_s (1 - [\mathscr{F}(\nu) + T\mathscr{W}(\nu) L_{\mathscr{U}}])\} \|\ell_s - z\|. \tag{3.9}
$$

Inductively, from (3.9), we have

$$
\|\ell_{s+1} - z\| \leq \|\ell_0 - z\| \prod_r^s \{1 - \delta_r \beta_r (1 - [\mathscr{F}(\nu) + T\mathscr{W}(\nu) L_{\mathscr{U}}])\}. \tag{3.10}
$$

Since $\delta_r, \beta_r \in [0,1]$ for all $r \in \mathbb{N}$, then from assumption (C_2) we get

$$
1 - \delta_r \beta_r (1 - [\mathscr{F}(\nu) + T\mathscr{W}(\nu) L_{\mathscr{U}}]) < 1.
$$

From classical analysis, we know that $1 - \ell \leq e^{-\ell}$ for all $\ell \in [0,1]$. Thus, (3.10) becomes

$$
\|\ell_{s+1} - z\| \leq \|\ell_0 - z\| e^{-\{1 - \delta_r \beta_r (1 - [\mathscr{F}(\nu) + T\mathscr{W}(\nu) L_{\mathscr{U}}])\} \sum_{r=0}^s \delta_r \beta_r}
$$

which yields $\lim_{s\to\infty} \|\ell_s - z\| = 0$. $\square$

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